Calculus – Convergence or Divergence – Test Summary

Test	Series	Convergence or Divergence
Divergence TEST	$\sum_{n=m}^{\infty} a_n, \ m \geq 0$	$if \lim_{n\to\infty} a_n \neq 0 \ or \ DNE \Rightarrow \sum_{n=m}^{\infty} a_n \Rightarrow divergent$
Absolutely Convergent $\sum_{n=m}^{\infty} \lvert a_n \rvert$	$\sum_{n=m}^{\infty} a_n, \ m \geq 0$	$if \sum_{n=m}^{\infty} \lvert a_n \rvert \ convergent \Rightarrow \sum_{n=m}^{\infty} a_n \ convergent$
Conditinally Convergent $\sum_{n=m}^{\infty} a_n$	$\sum_{n=m}^{\infty} a_n, \ m \geq 0$	$if \sum_{n=m}^{\infty} a_n \ convergent \ but \sum_{n=m}^{\infty} \lvert a_n \rvert \ divergent$
Geometric Series	$\sum_{n=0}^{\infty} ar^n \ , a \neq 0$	$Convergent \Rightarrow \sum_{n=0}^{\infty} ar^n = \frac{a}{1-r} \ if \lvert r \rvert < 1$ $Divergent \Rightarrow \sum_{n=0}^{\infty} ar^n \ if \lvert r \rvert \geq 1$
p – Series	$\sum_{n=m}^{\infty} \frac{1}{n^p}, \ m \geq 0$	$Convergent \ if \ p > 1$ $Divergent \ if \ p \leq 1$
Direct Comparison	$\sum_{n=m}^{\infty} a_n \ \& \sum_{n=m}^{\infty} b_n$ $0 \leq a_n \leq b_n, \forall n$	$if \sum_{n=m}^{\infty} b_n \ convergent \Rightarrow \sum_{n=m}^{\infty} a_n \ convergent$ $if \sum_{n=m}^{\infty} a_n \ divergent \Rightarrow \sum_{n=m}^{\infty} b_n \ divergent$
Limit Comparison	$\sum_{n=m}^{\infty} a_n \ \& \sum_{n=m}^{\infty} b_n$ $0 \leq a_n \leq b_n, \forall n$ $L = \lim_{n\to\infty} \frac{a^n}{b^n}$	$if \ 0 < L < \infty$ $\sum_{n=m}^{\infty} b_n \ convergent \Leftrightarrow \sum_{n=m}^{\infty} a_n \ convergent$ $\sum_{n=m}^{\infty} b_n \ divergent \Leftrightarrow \sum_{n=m}^{\infty} a_n \ divergent$ $if \ L = 0,$ $\sum_{n=m}^{\infty} b_n \ convergent \Rightarrow \sum_{n=m}^{\infty} a_n \ convergent$ $if \ L = \infty,$ $\sum_{n=m}^{\infty} b_n \ divergent \Rightarrow \sum_{n=m}^{\infty} a_n \ divergent$
Ratio Test	$\sum_{n=m}^{\infty} a_n, \ L = \lim_{n\to\infty} \frac{\lvert a_{n+1} \rvert}{\lvert a_n \rvert}$	$if \ L < 1 \Rightarrow Convergent \ Absolutely$ $if \ L > 1 \Rightarrow Divergent$ $if \ L = 1 \Rightarrow inconclusive$
Root Test	$\sum_{n=m}^{\infty} a_n, \ L = \lim_{n\to\infty} \sqrt[n]{\lvert a_n \rvert}$	$if \ L < 1 \Rightarrow Convergent$ $if \ L > 1 \Rightarrow Divergent$ $if \ L = 1 \Rightarrow inconclusive$
Integral Test	$\sum_{n=m}^{\infty} a_n, \ a_n = f(n), \forall n$	$if \int_m^{\infty} f(x) \ convergent \Rightarrow \sum_{n=m}^{\infty} a_n \ Convergent$ $if \int_m^{\infty} f(x) \ divergent \Rightarrow \sum_{n=m}^{\infty} a_n \ divergent$
Alternating Series	$\sum_{n=m}^{\infty} (-1)^n a_n, \ a_n > 0$	$if \ 0 \leq a_{n+1} \leq a_n \ \forall n, \quad L = \lim_{n\to\infty} a_n = 0$ $then \sum_{n=m}^{\infty} (-1)^n a_n \Rightarrow Conditionally \ Convergent$
Power Series $\sum_{n=0}^{\infty} c_n (x-a)^n \Rightarrow 3 \ Possibilities$	(i) *The series covnverges only when* $x = a. (R = 0)$ (ii) *The series covnverges for* $\forall x. \ (R = \infty)$ $(iii) \ \exists R > 0 \ convergent \ if \ \lvert x - a \rvert < R \ and \ divergent \ if \ \lvert x - a \rvert > R$ $IC \Rightarrow (a - R, a + R), (a - R, a + R], [a - R, a + R) \ or \ [a - R, a + R]$	