

Maclaurin Series of Hyperbolic Functions

$$f(x) = f(0) + \frac{f'(0)}{1!} \cdot x + \frac{f^{(2)}(0)}{2!} \cdot x^2 + \frac{f^{(3)}(0)}{3!} \cdot x^3 + \dots + \frac{f^{(n)}(0)}{n!} \cdot x^n + \dots$$

$\sinh x = \frac{e^x - e^{-x}}{2}$	$\cosh x = \frac{e^x + e^{-x}}{2}$	$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$
$\sinh x = -i \sin ix$	$\cosh x = \cos ix$	$\tanh x = -i \tan ix$
$\operatorname{csch} x = \frac{1}{\sinh x}$	$\operatorname{sech} x = \frac{1}{\cosh x}$	$\operatorname{coth} x = \frac{1}{\tanh x}$
$\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$	$\cosh^{-1} x = \ln(x + \sqrt{x^2 - 1})$	$\tanh^{-1} x = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right)$
$\cosh^2 x - \sinh^2 x = 1$	$1 - \tanh^2 x = \operatorname{sech}^2 x$	$1 - \operatorname{coth}^2 x = -\operatorname{csch}^2 x$
$\frac{d(\sinh x)}{dx} = \cosh x$	$\frac{d(\cosh x)}{dx} = \sinh x$	$\frac{d(\tanh x)}{dx} = \operatorname{sech}^2 x$
$\frac{d(\operatorname{csch} x)}{dx} = -\operatorname{csch} x \operatorname{coth} x$	$\frac{d(\operatorname{sech} x)}{dx} = -\operatorname{sech} x \tanh x$	$\frac{d(\operatorname{coth} x)}{dx} = -\operatorname{csch}^2 x$
$\frac{d(\sinh^{-1} x)}{dx} = \frac{1}{\sqrt{x^2 + 1}}$	$\frac{d(\cosh^{-1} x)}{dx} = \frac{1}{\sqrt{x^2 - 1}}$	$\frac{d(\tanh^{-1} x)}{dx} = \frac{1}{1 - x^2}, \quad x < 1$
$\frac{d(\operatorname{csch}^{-1} x)}{dx} = -\frac{1}{ x \sqrt{x^2 + 1}}$	$\frac{d(\operatorname{sech}^{-1} x)}{dx} = -\frac{1}{ x \sqrt{1 - x^2}}$	$\frac{d(\operatorname{coth}^{-1} x)}{dx} = \frac{1}{1 - x^2}, \quad x > 1$
First few terms of the Maclaurin Series	Maclaurin Series	Interval of Convergence
$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} \dots + \frac{x^{2n+1}}{(2n+1)!} + \dots$	$\sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$	All x
$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} \dots + \frac{x^{2n}}{(2n)!} + \dots$	$\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$	All x
$\sinh^{-1} x = x - \frac{1}{2 \cdot 3} x^3 + \frac{1 \cdot 3}{2 \cdot 4 \cdot 5} x^5 - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 7} x^7 + \dots$	$x + \sum_{n=1}^{\infty} (-1)^n \frac{1 \cdot 3 \cdot 5 \dots (2n-1) x^{2n+1}}{2 \cdot 4 \cdot 6 \dots (2n) \cdot (2n+1)}$	$ x < 1$
$\cosh^{-1} x = \ln(2x) - \frac{1}{2} \cdot \frac{1}{2x^2} - \frac{1 \cdot 3}{2 \cdot 4} \frac{1}{4x^4} + \dots$		$ x > 1$
$\tanh^{-1} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} \dots + \frac{x^{2n+1}}{2n+1} + \dots$	$\sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)}$	$ x \leq 1$
$\operatorname{coth}^{-1} x = \frac{1}{x} + \frac{1}{3x^3} + \frac{1}{5x^5} + \frac{1}{7x^7} \dots + \frac{1}{(2n+1)x^{2n+1}} + \dots$	$\sum_{n=0}^{\infty} \frac{x^{-(2n+1)}}{(2n+1)}$	$ x > 1$
$\tanh x = x - \frac{1}{3} x^3 + \frac{2}{15} x^5 - \frac{17}{315} x^7 + \frac{62}{2835} x^9 + \dots$		$ x < \frac{\pi}{2}$
$\operatorname{csch} x = \frac{1}{x} - \frac{1}{6} x + \frac{7}{360} x^3 - \frac{31}{15120} x^5 + \frac{127}{604800} x^7 + \dots$		$0 < x < \pi$
$\operatorname{sech} x = 1 - \frac{1}{2} x^2 + \frac{5}{24} x^4 - \frac{61}{720} x^6 + \frac{277}{8064} x^8 + \dots$		$ x < \frac{\pi}{2}$
$\operatorname{coth} x = \frac{1}{x} + \frac{1}{3} x - \frac{1}{45} x^3 + \frac{2}{945} x^5 - \frac{1}{4725} x^7 - \dots$		$0 < x < \pi$
$\operatorname{sech}^{-1} x = \ln \frac{2}{x} - \frac{1}{2} \cdot \frac{x^2}{2} - \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{x^4}{4} - \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \cdot \frac{x^6}{6} \dots$		$0 < x < 1$
$\operatorname{csch}^{-1} x = \ln \frac{2}{x} + \frac{1}{2} \cdot \frac{x^2}{2} + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{x^4}{4} + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \cdot \frac{x^6}{6} \dots$		$0 < x < 1$
$\operatorname{csch}^{-1} x = \frac{1}{x} - \frac{1}{2} \cdot \frac{1}{3x^3} - \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{1}{5x^5} - \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \cdot \frac{1}{7x^7} + \dots$		$ x > 1$
$(1+x)^p = 1 + p \cdot \frac{x}{1!} + \frac{p \cdot (p-1)}{2!} x^2 + \dots + \frac{p(p-1)(p-2) \dots (p-n+1)}{n!} x^n + \dots$		$ x < 1$