Useful Series

$S_n = 1+2+3+4+5+\cdots+n. \quad = \sum_{i=1}^{n} i = \frac{n}{2}(n+1)$	
$S_n = 1^2+2^2+3^2+4^2+5^2+\cdots+n^2 = \sum_{i=1}^{n} i^2 = \frac{n}{6}(n+1)(2n+1)$	
$S_n = 1^3+2^3+3^3+4^3+5^3+\cdots+n^3 = \sum_{i=1}^{n} i^3 = \left[\frac{n}{2}(n+1)\right]^2 = \frac{n^2}{4}(n+1)^2$	
$S_n = 1^4+2^4+3^4+4^4+5^4+\cdots+n^4 = \sum_{i=1}^{n} i^4 = \frac{1}{30}n(n+1)(2n+1)(3n^2+3n-1)$	
$S_n = 1^5+2^5+3^5+4^5+5^5+\cdots+n^5 = \sum_{i=1}^{n} i^5 = \frac{1}{12}n^2(n+1)^2(2n^2+2n-1)$	
$S_n = a_1+(a_1+d)+(a_1+2d)+\cdots+[a_1+(n-1)d] = \frac{n}{2}(a_1+a_n) = \frac{n}{2}\{2a_1+(n-1)d\}$	
$S_n = a_1+a_1r+a_1r^2+\cdots+a_1r^{n-1} = \sum_{n=1}^{n} a_1r^{n-1} = \frac{a_1(1-r^n)}{1-r} \quad for\ r \neq 1$	
$\frac{1+3+5+\cdots+(2n-1)}{(2n+1)+(2n+3)+(2n+5)+\cdots+(4n-1)} = \frac{1}{3}$	$1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{(-1)^{n-1}}{2n-1}+\cdots = \frac{\pi}{4}$
$1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\cdots+\frac{1}{n^2}+\cdots = \frac{\pi^2}{6}$	$1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\cdots+\frac{1}{(2n-1)^2}+\cdots = \frac{\pi^2}{8}$
$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$; $\frac{1}{e} = \lim_{n\to\infty}\left(1-\frac{1}{n}\right)^n$	$\sum_{n=1}^{\infty}\frac{1}{n(n+1)} = 1$
$e^x = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots+\frac{x^n}{n!}+\cdots = \sum_{n=0}^{\infty}\frac{x^n}{n!}$	$\sum_{n=1}^{\infty}\frac{1}{n(n+1)(n+2)\ldots(n+p)} = \frac{1}{p\cdot p!}$
$a\cdot(a+1)\cdot(a+2)\ldots(a+n-1) = \frac{(a+n-1)!}{(a-1)!} = \frac{\Gamma(a+n)}{\Gamma(a)}$	
$1+x+x^2+x^3+x^4+\cdots+x^n+\cdots = \frac{1}{1-x} \qquad \lvert x\rvert < 1$	
$a_1+(a_1+d)x+(a_1+2d)x^2+\cdots+[a_1+(n-1)d]x^{n-1} = \frac{a_1}{1-x}+\frac{d\cdot x}{(1-x)^2} \qquad \lvert x\rvert < 1$	
$(1+x)^a = 1+a\cdot\frac{x}{1!}+\frac{a\cdot(a-1)}{2!}x^2+\cdots+\binom{a}{n}x^n+\cdots = \sum_{n=0}^{\infty}\binom{a}{n}x^n \qquad \lvert x\rvert < 1$	
$(1+x)^{-a} = 1-\binom{a}{1}\cdot x+\binom{a+1}{2}x^2-\binom{a+2}{3}x^3+\cdots+(-1)^{n-1}\binom{a+n-1}{n}x^n+\cdots \qquad \lvert x\rvert < 1$	
$(1-x)^{-a} = 1+\binom{a}{1}\cdot x+\binom{a+1}{2}x^2+\binom{a+2}{3}x^3+\cdots+\binom{a+n-1}{n}x^n+\cdots \qquad \lvert x\rvert < 1$	
$f(x) = f(0)+\frac{f'(0)}{1!}\cdot x+\frac{f^{(2)}(0)}{2!}\cdot x^2+\frac{f^{(3)}(0)}{3!}\cdot x^3+\cdots+\frac{f^{(n)}(0)}{n!}\cdot x^n+\cdots$	
$f(x) = f(a)+\frac{f'(a)}{1!}\cdot(x-a)+\frac{f^{(2)}(a)}{2!}\cdot(x-a)^2+\frac{f^{(3)}(a)}{3!}\cdot(x-a)^3+\cdots+\frac{f^{(n)}(a)}{n!}\cdot(x-a)^n+\cdots$	
$\phi = \sqrt{1+\sqrt{1+\sqrt{1+\cdots}}} = \sqrt{1+\phi} = \frac{1+\sqrt{5}}{2}$	$n! \approx \sqrt{2\pi e}\left(\frac{n}{e}\right)^{n+\frac{1}{2}} \quad for\ large\ n$