Trigonometry Formula Sheet

$180^{\circ} = \pi$ *radians* | *DMS*: $1^{\circ} = 60'$ *and* $1' = 60''$

Trigonometric Functions

$$\sin\theta = \frac{opp}{hyp} \qquad \cos\theta = \frac{adj}{hyp} \qquad \tan\theta = \frac{opp}{adj}$$

$$\sin\theta \cdot \csc\theta = 1 \qquad \cos\theta \cdot \sec\theta = 1 \qquad \tan\theta \cdot \cot\theta = 1$$

Sum and Difference Formulas

$$\sin(A \pm B) = \sin(A)\cos(B) \pm \cos(A)\sin(B)$$

$$\cos(A \pm B) = \cos(A)\cos(B) \mp \sin(A)\sin(B)$$

$$\tan(A \pm B) = \frac{\tan(A) \pm \tan(B)}{1 \mp \tan(A)\tan(B)}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Linear Velocity(v) and Angular Speed(ω)

$$s = r\theta, \qquad A = \frac{1}{2}r^2\theta, \qquad v = r\omega, \qquad \omega = \frac{\theta}{t}$$

Cofunction Identities

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta \Leftrightarrow \sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta \Leftrightarrow \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$

$$\sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta \Leftrightarrow \csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$$

Sum to Product Formulas

$$\sin(A) \pm \sin(B) = 2\sin\left(\frac{A \pm B}{2}\right)\cos\left(\frac{A \mp B}{2}\right)$$

$$\cos(A) + \cos(B) = 2\cos\left(\frac{A + B}{2}\right)\cos\left(\frac{A - B}{2}\right)$$

$$\cos(A) - \cos(B) = 2\sin\left(\frac{A + B}{2}\right)\sin\left(\frac{B - A}{2}\right)$$

Odd and Even Identities

$\sin(-\theta) = -\sin\theta$. $\cos(-\theta) = \cos\theta$

$\csc(-\theta) = -\csc\theta$ $\sec(-\theta) = \sec\theta$

$\tan(-\theta) = -\tan\theta$ $\cot(-\theta) = -\cot\theta$

Product to Sum Formulas

$$2\sin(A)\cos(B) = \sin(A + B) + \sin(A - B)$$

$$2\cos(A)\cos(B) = \cos(A + B) + \cos(A - B)$$

$$2\sin(A)\sin(B) = \cos(A - B) - \cos(A + B)$$

Law of Sines and Cosines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cdot \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cdot \cos C$$

Double − Angle Formulas

$$\sin(2A) = 2\sin(A)\cos(A)$$

$$\cos(2A) = \cos^2A - \sin^2A = 2\cos^2A - 1 = 1 - 2\sin^2A$$

$$\tan(2A) = \frac{2\tan(A)}{1 - \tan^2A}$$

Dot Product

If $\boldsymbol{u} = < u_1, u_2, u_3 >$ *and* $\boldsymbol{v} = < v_1, v_2, v_3 >$, *then*

$$\boldsymbol{u} \cdot \boldsymbol{v} = |u||\boldsymbol{v}|\cos\theta = u_1v_1 + u_2v_2 + u_3v_3$$

Half Angle Formulas

$$\sin^2\left(\frac{A}{2}\right) = \frac{1 - \cos(A)}{2} \qquad \cos^2\left(\frac{A}{2}\right) = \frac{1 + \cos(A)}{2}$$

$$\tan\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{\sin(A)}{1 + \cos(A)} = \frac{1 - \cos(A)}{\sin(A)}$$

Vector Projection

$$\boldsymbol{proj_v}u = \frac{(\boldsymbol{u} \cdot \boldsymbol{v})}{(\boldsymbol{v} \cdot \boldsymbol{v})}\boldsymbol{v} \qquad (\boldsymbol{u} \text{ component } \parallel \text{ to } \boldsymbol{v})$$

$$\boldsymbol{u} - \boldsymbol{proj_v}u \qquad (\boldsymbol{u} \text{ component } \perp \text{ to } \boldsymbol{v})$$

Triple Angle Formulas

$$\sin(3A) = 3\sin(A) - 4\sin^3(A) \qquad \cos(3A) = 4\cos^3 A - 3\cos(A)$$

Polar Coordinates (r, θ)

$$(\boldsymbol{x}, \boldsymbol{y}) = (\boldsymbol{r}\cos\boldsymbol{\theta}, \boldsymbol{r}\sin(\boldsymbol{\theta})), \tan\boldsymbol{\theta} = \frac{\boldsymbol{y}}{\boldsymbol{x}}, \boldsymbol{x} \neq \boldsymbol{0}$$

Distanance between $P_1(r_1, \theta_1)$ & $P_2(r_2, \theta_2)$

$$d(P_1, P_2) = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}$$

Polar Axis ($x - axis$) Symmetry : *If replace (r, θ) by $(r, -\theta)$ gives and equivalent equation.*

$\theta = \frac{\pi}{2}$ ($y - axis$) Symmetry : *If replacing (r, θ) by $(-r, -\theta)$ or $(r, \pi - \theta)$ gives an equivalent equation.*

Pole (Origin) Symmetry : *If replacing (r, θ) by $(-r, \theta)$ or $(r, \pi + \theta)$ gives and equivalent equation.*

Polar Equations of Lines

$$\boldsymbol{y} = \boldsymbol{mx} = (\tan\boldsymbol{\alpha})\boldsymbol{x} \Leftrightarrow \boldsymbol{\theta} = \boldsymbol{\alpha}$$

$$\boldsymbol{ax} + \boldsymbol{by} = \boldsymbol{c} \Leftrightarrow \boldsymbol{r} = \frac{\boldsymbol{c}}{\boldsymbol{a}\cos\boldsymbol{\theta} + \boldsymbol{b}\sin(\boldsymbol{\theta})}, \boldsymbol{c} \neq \boldsymbol{0}$$

Periodic Functions : $f(t + P) = f(t)$

$$S(t) = A\sin(\omega t - \phi) + C \text{ and } C(t) = A\cos(\omega t - \phi) + C$$

$$Amplitude = A; \quad P = \frac{2\pi}{\omega}; \quad f = \frac{\omega}{2\pi}; \quad Phase\ Shift = \frac{\emptyset}{\omega}$$

Complex Numbers

$\boldsymbol{Z} = \boldsymbol{x} + \boldsymbol{iy}$ (*Rectangular form*), *where* $(\boldsymbol{i} = \sqrt{-1})$

$\boldsymbol{Z} = \boldsymbol{r}(\cos\boldsymbol{\theta} + \boldsymbol{i}\sin\boldsymbol{\theta}) = \boldsymbol{rcis\theta}$ (*Polar form*)

$\boldsymbol{Z} = \boldsymbol{re^{i\theta}}$ (*Euler form*)

Area of a Triangle

$$A = \sqrt{s(s - a)(s - b)(s - c)}, \text{ where } s = \frac{1}{2}(a + b + c)$$

De Moivre'sTheorem

$$Z^n = [r(\cos\theta + i\sin\theta)]^n = r^n\left(cis(n\theta)\right) = r^ne^{i(n\theta)}$$

Complex Number Product Theorem

$$Z_1 \cdot Z_2 = (r_1cis\theta_1) \cdot (r_2cis\theta_2) = r_1r_2cis(\theta_1 + \theta_2)$$

Complex Number Quotient Theorem

$$\frac{Z_1}{Z_2} = \frac{r_1cis\theta_1}{r_2cis\theta_2} = \frac{r_1}{r_2}cis(\theta_1 - \theta_2)$$

Cross Product

$$\boldsymbol{u} \times \boldsymbol{v} = |u||\boldsymbol{v}|\sin\theta\vec{\boldsymbol{n}}$$

$$\boldsymbol{u} \times \boldsymbol{v} = \langle u_2v_3 - u_3v_2, u_3v_1 - u_1v_3, u_1v_2 - u_2v_1 \rangle$$

n^{th} Root of a Complex Number

$$w_k = \sqrt[n]{Z} = \sqrt[n]{r}cis\left(\frac{\theta + 2\pi k}{n}\right) \Rightarrow w_0, w_1, \ldots\ldots w_{n-1}$$