Advanced Algebra Formula Sheet

Quadratic Function

$$f(x) = ax^2 + bx + c = a(x-h)^2 + h$$
$$\Delta = discriminat = b^2 - 4ac$$
$$Vertex\ \ of\ a\ Parabola : (h,k)$$
$$(h,k) = \left(-\frac{b}{2a}, f(-\frac{b}{2a})\right) = \left(-\frac{b}{2a}, \frac{-\Delta}{4a}\right)$$

Factoring

$$(a^2 - b^2) = (a-b)(a+b)$$
$$(a^4 - b^4) = (a-b)(a+b)(a^2+b^2)$$
$$(a^3 + b^3) = (a+b)(a^2 - ab + b^2)$$
$$(a^3 - b^3) = (a-b)(a^2 + ab + b^2)$$

Absolute Value

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

n^{th} root of x^n

$$\sqrt[n]{x^n} = \begin{cases} |x| \ \ if\ n\ is\ even \\ \ x\ \ if\ n\ is\ odd \end{cases}$$

Quadratic Formula

$$If\ \ ax^2 + bx + c = 0,\ \ a \neq 0,$$
$$\Rightarrow\ \ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{\Delta}}{2a}$$

Midpoint & Distance

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Slope (m)

$$P_1(x_1, y_1)\ \&\ P_2(x_2, y_2)$$
$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Arithmetic Series

$$a_n = a_1 + (n-1)d$$
$$S_n = \frac{n}{2}(a_1 + a_n) = \frac{n}{2}[2a_1 + (n-1)d]$$

Geometric Series

$$a_n = a_1 r^{n-1}$$
$$S_n = \frac{a_1(1 - r^n)}{1 - r},\ \ S_\infty = \frac{a_1}{1 - r}\ \ where\ |r| < 1$$

Complex Numbers

$$a + ib\ \ is\ a\ complex\ number$$
$$Where\ \ i = \sqrt{-1}\ \ \Rightarrow\ \ i^2 = -1$$
$$\sqrt{-b} = i\sqrt{b}\ \ \ \ for\ \ b > 0$$

Inverse Functon

$$(f \circ f^{-1})(x) = f(f^{-1}(x)) = x$$
$$(f^{-1} \circ f)(x) = f^{-1}(f(x)) = x$$

Factorial

$$n! = n \cdot (n-1) \cdot \ldots 3 \cdot 2.1$$

Permutation & Combination

$${}^nP_r = \frac{n!}{(n-r)!}$$
$${}^nC_r = \binom{n}{r} = \frac{n!}{(n-r)!\,r!}$$

Pascal Triangle

$\binom{0}{0}$	1
$\binom{1}{0}\binom{1}{1}$	1 1
$\binom{2}{0}\binom{2}{1}\binom{2}{2}$	1 2 1
$\binom{3}{0}\binom{3}{1}\binom{3}{2}\binom{3}{3}$	1 3 3 1
$\binom{4}{0}\binom{4}{1}\binom{4}{2}\binom{4}{3}\binom{4}{4}$	1 4 6 4 1

Binomial Theorem

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^2 b + \cdots \binom{n}{r}a^{n-r}b^r + \cdots + \binom{n}{r}b^n$$
$$where\ \binom{n}{r} = \frac{n!}{(n-r)!\,r!}\ \ \ \&\ \ \binom{n}{r-1} + \binom{n}{r} = \binom{n+1}{r}$$
$$r^{th}\ term\ of\ a\ binomial\ expansion = \binom{n}{r-1}a^{n-r+1}b^{r-1}$$

E. g. 1: $(a+b)^3 = \binom{3}{0}a^3b^0 + \binom{3}{1}a^2b^1 + \binom{3}{2}a^1b^2 + \binom{3}{3}a^0b^3 = a^3 + 3a^2b + 3ab^2 + b^3$

E. g. 2: $(a+b)^4 = \binom{4}{0}a^4b^0 + \binom{4}{1}a^3b^1 + \binom{4}{2}a^2b^2 + \binom{4}{3}a^1b^3 + \binom{4}{4}a^0b^4 = a^4 + 4a^3b^1 + 6a^2b^2 + 4ab^3 + b^4$

Exponential Properties ($a \neq 0, b \neq 0$)

$b^0 = 1$	$b^m b^n = b^{m+n}$
$\frac{b^m}{b^n} = b^{m-n}$	$(b^m)^n = b^{mn}$
$b^{-n} = \frac{1}{b^n}$	$(ab)^n = a^n b^n$
$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$
$b^{\frac{1}{n}} = \sqrt[n]{b}$	$b^{\frac{m}{n}} = \sqrt[n]{b^m} = (\sqrt[n]{b})^m$

Forms for the Equation of a line

$y = mx + b$	✓ $Slope\ intercept\ form$: $m\ \&\ P(0, b)$
$y - y_1 = m(x - x_1)$	✓ $Point\ slope\ form$: $P(x_1, y_1)\ \&\ m$
$\frac{x}{a} + \frac{y}{b} = 1$	✓ $Intercept\ form$: $P_1(a, 0)\ \&\ P_2(0, b)$
$Ax + By = C$	✓ $Standard\ form$: $A, B\ \&\ C\ are\ integers$
$x = a$ $y = b$	✓ $Vertical\ line$: $Contains\ \ P(a, 0)$ ✓ $Horizontal\ line$: $Contains\ P(0, b)$

Logarithm Properties ($b \neq 0$, $b \neq 1$)

$$b^E = N \iff \log_b N = E$$
$$e^E = N \iff E = \ln N$$
$$\log_b x + \log_b y = \log_b xy$$
$$\log_b x - \log_b y = \log_b\left(\frac{x}{y}\right)$$

$\log_b b = 1$	$\log_b x^n = n\log_b x$	$b^{\log_b N} = N$	$\ln e = 1$	$\log_b a = \frac{\log(a)}{\log(b)}$	$\ln(a) = \frac{1}{\log_a e}$

$\log_b a = \frac{1}{\log_a b}$	$\log_b \frac{1}{a} = -\log_b a$	$\log_{\left(\frac{1}{a}\right)} b = -\log_a b$	$\log_a b \cdot \log_b c = \log_a c$	$\log_{a^m}(a^n) = \frac{n}{m}, m \neq 0$

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1
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1 2 1
1 3 3 1
1 4 6 4 1

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$\log_b b = 1$	$\log_b x^n = n\log_b x$	$b^{\log_b N} = N$	$\ln e = 1$	$\log_b a = \frac{\log(a)}{\log(b)}$	$\ln(a) = \frac{1}{\log_a e}$

$\log_b a = \frac{1}{\log_a b}$	$\log_b \frac{1}{a} = -\log_b a$	$\log_{\left(\frac{1}{a}\right)} b = -\log_a b$	$\log_a b \cdot \log_b c = \log_a c$	$\log_{a^m}(a^n) = \frac{n}{m}, m \neq 0$