Differentiation Rules

$$f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h} \quad or \quad f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

$$\frac{d[c]}{dx}=0$$

$$\frac{d[u^n]}{dx}=nu^{n-1}u'$$

$$[c_1u\pm c_2v]'=c_1u'\pm c_2v'$$

$$[uv]'=u'v+uv'$$

$$\left[\frac{u}{v}\right]'=\frac{u'v-uv'}{v^2}$$

$$(fog)'(u)=[f(g(u))]'=f'\big(g(u)\big)g'(u)u'$$

$$[e^u]'=e^uu'$$

$$[\ln u]'=\frac{u'}{u}$$

$$[b^u]'=(\ln b)b^uu'$$

$$[\log_b u]'=\frac{u'}{(\ln b)u}$$

$$[\sin u]'=(\cos u)u'$$

$$[\cos u]'=-(\sin u)u'$$

$$[\tan u]'=(\sec^2 u)u'$$

$$[\cot u]'=-(\csc^2 u)u'$$

$$[\sec u]'=(\sec u\tan u)u'$$

$$[\csc u]'=-(\csc u\cot u)u'$$

$$[\sin^{-1}u]'=\frac{u'}{\sqrt{1-u^2}}$$

$$[\cos^{-1}u]'=\frac{-u'}{\sqrt{1-u^2}}$$

$$[\tan^{-1}u]'=\frac{u'}{1+u^2}$$

$$[\cot^{-1}u]'=\frac{-u'}{1+u^2}$$

$$[\sec^{-1}u]'=\frac{u'}{|u|\sqrt{u^2-1}}$$

$$[\csc^{-1}u]'=\frac{-u'}{|u|\sqrt{u^2-1}}$$

$$\sinh u=\frac{e^u-e^{-u}}{2}$$

$$\cosh u=\frac{e^u+e^{-u}}{2}$$

$$\tanh u=\frac{e^u-e^{-u}}{e^u+e^{-u}}$$

$$[\sinh u]'=(\cosh u)u'$$

$$[\cosh u]'=(\sinh u)u'$$

$$[\tanh u]'=(\text{sech}^2 u)u'$$

$$[\coth u]'=-(\text{csch}^2 u)u'$$

$$[\text{sech}\, u]'=-(\text{sech}\, u\tanh u)u'$$

$$[\text{csch}\, u]'=-(\text{csch}\, u\coth u)u'$$

$$\sinh^{-1}u=\ln\left(u+\sqrt{u^2+1}\right)$$

$$\cosh^{-1}u=\ln\left(u+\sqrt{u^2-1}\right),\ u>1$$

$$\tanh^{-1}u=\frac{1}{2}\ln\left(\frac{1+u}{1-u}\right),\ -1<u<1$$

$$[\sinh^{-1}u]'=\frac{u'}{\sqrt{u^2+1}}$$

$$[\cosh^{-1}u]'=\frac{u'}{\sqrt{u^2-1}}$$

$$[\tanh^{-1}u]'=\frac{u'}{1-u^2}$$

$$[\coth^{-1}u]'=\frac{u'}{1-u^2}$$

$$[\text{sech}^{-1}u]'=\frac{-u'}{u\sqrt{1-u^2}}$$

$$[\text{csch}^{-1}u]'=\frac{-u'}{|u|\sqrt{1+u^2}}$$

$x=f(t)$ and $y=g(t)$

$$\frac{dy}{dx}=\frac{dy}{dt}\Big/\frac{dx}{dt}=\frac{g'(t)}{f'(t)}$$

If f is a One to One function and $f'\left(f^{-1}(b)\right)\neq 0$, at (a,b)

$$\left(f^{-1}\right)'(b)=\frac{1}{f'(f^{-1}(b))}=\frac{1}{f'(a)}$$

$x=r(\theta)\cos\theta$ and $y=r(\theta)\sin\theta$

$$\frac{dy}{dx}=\frac{dy}{d\theta}\Big/\frac{dx}{d\theta}=\frac{\left(\frac{dr}{d\theta}\right)\sin\theta+r\cos\theta}{\left(\frac{dr}{d\theta}\right)\cos\theta-r\sin\theta}$$

$$f_x(a,b)=\lim_{h\to 0}\frac{f(a+h,b)-f(a,b)}{h}$$

$$\nabla f(x,y)=\langle f_x,f_y\rangle$$

$$f_y(a,b)=\lim_{h\to 0}\frac{f(a,b+h)-f(a,b)}{h}$$

The directional derivative of $f(x,y)$ at (x_0,y_0) in the direction $u=\langle a,b\rangle$

$$D_uf(x_0,y_0)=\lim_{h\to 0}\frac{f(x_0+ah,y_0+bh)-f(x_0,y_0)}{h}$$

$$D_uf(x_0,y_0)=af_x(x_0,y_0)+bf_y(x_0,y_0)=\nabla f(x_0,y_0)\cdot u$$