Answers and Rubrics (AB)

Answers to Multiple-Choice Questions

1.	B
2.	B
3.	C
4.	A
5.	A
6.	B
7.	D
8.	D
9.	B
10.	A
11.	A
12.	A
13.	D
14.	C
15.	D
16.	B

Rubrics for Free-Response Questions

Question 1

<table>
<tr><th>Solutions</th><th>Point Allocation</th></tr>
<tr><td>

(a) $\frac{dV}{dt} = \frac{1}{3}\pi 3h^2 \frac{dh}{dt} = \pi h^2 \frac{dh}{dt}$

At $t = 0$,

$\frac{dV}{dt} = \pi(25)^2(-4) = -2500\pi = -7853.982$ (or -7853.981) cubic meters per hour.

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$2: \begin{cases} 1: \frac{dV}{dt} \\ 1: \text{answer with units} \end{cases}$

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(b) The absolute minimum must be at a critical point or an endpoint.

$h'(t) = 0$ when $t = 6.261$.

$h(t) = 25 + \int_0^t h'(x)\,dx$

<table>
<tr><th>t</th><th>$h(t)$</th></tr>
<tr><td>0</td><td>25</td></tr>
<tr><td>6.261</td><td>16.33873</td></tr>
<tr><td>24</td><td>34.56246</td></tr>
</table>

The minimum height is 16.339 (or 16.338) meters.

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$4: \begin{cases} 1: \text{considers } h'(t) = 0 \\ 1: \text{Fundamental Theorem} \\ \quad \text{of Calculus} \\ 1: \text{absolute minimum value} \\ 1: \text{justification} \end{cases}$

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(c) $h(16) = 25 + \int_0^{16} h'(t)\,dt = 23.49607$

$h'(16) = 1.19562$

An equation for the tangent line is

$y = 1.196(t - 16) + 23.496$.

$y = 25$ when $t = 17.258$ (or 17.257).

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$3: \begin{cases} 1: h(16) \\ 1: \text{tangent line equation} \\ 1: \text{answer} \end{cases}$

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Question 2

Solutions	Point Allocation
(a) g has a relative maximum at $x=-2$ since $g'=f$ changes sign from positive to negative at $x=-2$.	1 : answer with justification
(b) The graph of g is concave up for $-1<x<1$ and $2<x<3$ because $g'=f$ is increasing on those intervals.	$2:\begin{cases}1:\text{answer}\\1:\text{reason}\end{cases}$
(c) Because g is continuous at $x=0$, $\lim\limits_{x\to0} g(x)=g(0)$. $g(3)=g(0)+\int_0^3 f(x)\,dx$ $g(0)=g(3)-\int_0^3 f(x)\,dx=7-(5+3)=-1$ $\lim\limits_{x\to0} g(x)+1=0$ and $\lim\limits_{x\to0} 2x=0$. Using L'Hospital's Rule, $\lim\limits_{x\to0}\frac{g(x)+1}{2x}=\lim\limits_{x\to0}\frac{g'(x)}{2}=\lim\limits_{x\to0}\frac{f(x)}{2}=\frac{f(0)}{2}=0$	$3:\begin{cases}1:\ g(0)\\1:\text{L'Hospital's Rule}\\1:\text{answer}\end{cases}$
(d) $\int_{-2}^{1} h(x)\,dx=\int_{-2}^{1}\left(3f(2x+1)+4\right)dx=3\int_{-2}^{1} f(2x+1)\,dx+\int_{-2}^{1}4\,dx$ Let $u=2x+1$. Then $du=2dx$ and $3\int_{-2}^{1} f(2x+1)\,dx+\int_{-2}^{1}4\,dx=\frac{3}{2}\int_{-3}^{3} f(u)\,du+12$ $=\frac{3}{2}(5-4+5+3)+12=25.5$	$3:\begin{cases}2:\text{Fundamental Theorem}\\\quad\text{of Calculus}\\1:\text{answer}\end{cases}$

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