AP Calculus BC Sample Exam Questions

Multiple Choice: Section I, Part A

A calculator may not be used on questions on this part of the exam.

1. The position of a particle moving in the xy-plane is given by the parametric equations $x(t) = \frac{6t}{t+1}$ and $y(t) = \frac{-8}{t^2+4}$. What is the slope of the line tangent to the path of the particle at the point where $t = 2$?

(A) $\frac{1}{2}$

(B) $\frac{2}{3}$

(C) $\frac{3}{4}$

(D) $\frac{4}{3}$

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
LO 2.1C: Calculate derivatives.	**EK 2.1C7:** *(BC)* Methods for calculating derivatives of real-valued functions can be extended to vector-valued functions, parametric functions, and functions in polar coordinates.	**MPAC 3:** Implementing algebraic/computational processes **MPAC 2:** Connecting concepts

2. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = 1 + 2y$ with the initial condition $f(0) = 1$. What is the approximation for $f(1)$ if Euler's method is used, starting at $x = 0$ with a step size of 0.5?

(A) 2.5

(B) 3.5

(C) 4.0

(D) 5.5

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
LO 2.3F: Estimate solutions to differential equations.	**EK 2.3F2:** *(BC)* For differential equations, Euler's method provides a procedure for approximating a solution or a point on a solution curve.	**MPAC 3:** Implementing algebraic/computational processes **MPAC 2:** Connecting concepts

3. For what value of k, if any, is $\int_0^{\infty} kxe^{-2x}\,dx = 1$?

(A) $\frac{1}{4}$

(B) 1

(C) 4

(D) There is no such value of k.

Learning Objectives	Essential Knowledge	Mathematical Practices for AP Calculus
LO 3.2D: *(BC)* Evaluate an improper integral or show that an improper integral diverges.	**EK 3.2D2:** *(BC)* Improper integrals can be determined using limits of definite integrals.	**MPAC 3:** Implementing algebraic/computational processes
LO 3.3B(b): Evaluate definite integrals.	**EK 3.3B5:** Techniques for finding antiderivatives include algebraic manipulation such as long division and completing the square, substitution of variables, *(BC)* integration by parts, and nonrepeating linear partial fractions.	**MPAC 1:** Reasoning with definitions and theorems

4. The Taylor series for a function f about $x = 0$ converges to f for $-1 \le x \le 1$. The nth-degree Taylor polynomial for f about $x = 0$ is given by $P_n(x) = \sum_{k=1}^{n} (-1)^k \frac{x^k}{k^2 + k + 1}$. Of the following, which is the smallest number M for which the alternating series error bound guarantees that $\left| f(1) - P_4(1) \right| \le M$?

(A) $\frac{1}{5!} \cdot \frac{1}{31}$

(B) $\frac{1}{4!} \cdot \frac{1}{21}$

(C) $\frac{1}{31}$

(D) $\frac{1}{21}$

Learning Objectives	Essential Knowledge	Mathematical Practices for AP Calculus
LO 4.1B: Determine or estimate the sum of a series.	**EK 4.1B2:** If an alternating series converges by the alternating series test, then the alternating series error bound can be used to estimate how close a partial sum is to the value of the infinite series.	**MPAC 1:** Reasoning with definitions and theorems **MPAC 5:** Building notational fluency
LO 4.2B: Write a power series representing a given function.	**EK 4.2B4:** A Taylor polynomial for $f(x)$ is a partial sum of the Taylor series for $f(x)$.	

x	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$
0	3	–2	1	4
1	2	–3	3	–2
2	–1	1	4	5

5. Selected values of a function *f* and its first three derivatives are indicated in the table above. What is the third-degree Taylor polynomial for *f* about $x = 1$?

(A) $2 - 3x + \frac{3}{2}x^2 - \frac{1}{3}x^3$

(B) $2 - 3(x-1) + \frac{3}{2}(x-1)^2 - \frac{1}{3}(x-1)^3$

(C) $2 - 3(x-1) + \frac{3}{2}(x-1)^2 - \frac{2}{3}(x-1)^3$

(D) $2 - 3(x-1) + 3(x-1)^2 - 2(x-1)^3$

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
LO 4.2A: Construct and use Taylor polynomials.	**EK 4.2A1:** The coefficient of the *n*th-degree term in a Taylor polynomial centered at $x = a$ for the function f is $\frac{f^{(n)}(a)}{n!}$.	**MPAC 1:** Reasoning with definitions and theorems **MPAC 4:** Connecting multiple representations

6. Which of the following statements about the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{1+\sqrt{n}}$ is true?

(A) The series converges absolutely.

(B) The series converges conditionally.

(C) The series converges but neither conditionally nor absolutely.

(D) The series diverges.

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
LO 4.1A: Determine whether a series converges or diverges.	**EK 4.1A4:** A series may be absolutely convergent, conditionally convergent, or divergent.	**MPAC 1:** Reasoning with definitions and theorems **MPAC 5:** Building notational fluency

Multiple Choice: Section I, Part B

A graphing calculator is required for some questions on this part of the exam.

7. At time $t \geq 0,$ a particle moving in the xy-plane has velocity vector given by $v(t) = \left\langle 4e^{-t}, \sin\left(1+\sqrt{t}\right)\right\rangle$. What is the total distance the particle travels between $t = 1$ and $t = 3$?

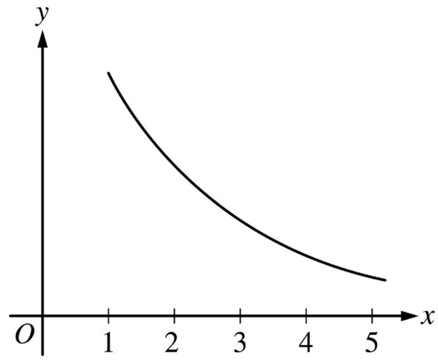
(A) 1.861

(B) 1.983

(C) 2.236

(D) 4.851

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
LO 3.4C: Apply definite integrals to problems involving motion.	**EK 3.4C2:** *(BC)* The definite integral can be used to determine displacement, distance, and position of a particle moving along a curve given by parametric or vector-valued functions.	**MPAC 2:** Connecting concepts **MPAC 3:** Implementing algebraic/computational processes



Graph of g

8. For $x \geq 1,$ the continuous function g is decreasing and positive. A portion of the graph of g is shown above. For $n \geq 1$, the nth term of the series $\sum_{n=1}^{\infty} a_n$ is defined by $a_n = g(n)$. If $\int_1^{\infty} g(x)\,dx$ converges to 8, which of the following could be true?

(A) $\sum_{n=1}^{\infty} a_n = 6$

(B) $\sum_{n=1}^{\infty} a_n = 8$

(C) $\sum_{n=1}^{\infty} a_n = 10$

(D) $\sum_{n=1}^{\infty} a_n$ diverges

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
LO 4.1A: Determine whether a series converges or diverges.	**EK 4.1A6:** In addition to examining the limit of the sequence of partial sums of the series, methods for determining whether a series of numbers converges or diverges are the *n*th term test, the comparison test, the limit comparison test, the integral test, the ratio test, and the alternating series test.	**MPAC 1:** Reasoning with definitions and theorems **MPAC 4:** Connecting multiple representations

Free Response: Section II, Part A

A graphing calculator is required for problems on this part of the exam.

1. At time $t \geq 0,$ the position of a particle moving along a curve in the xy-plane is $(x(t), y(t)),$ where $\frac{dx}{dt} = t - 5\cos t$ and $\frac{dy}{dt} = 6\cos(1 + \sin t)$. At time $t = 3$, the particle is at position $(-1, 2)$.

(a) Write an equation for the line tangent to the path of the particle at time $t = 3$.

(b) Find the time t when the line tangent to the path of the particle is vertical. Is the direction of motion of the particle up or down at that moment? Give a reason for your answer.

(c) Find the y-coordinate of the particle's position at time $t = 0$.

(d) Find the total distance traveled by the particle for $0 \leq t \leq 3$.

Learning Objectives	Essential Knowledge	Mathematical Practices for AP Calculus
LO 2.1C: Calculate derivatives.	**EK 2.1C7:** *(BC)* Methods for calculating derivatives of real-valued functions can be extended to vector-valued functions, parametric functions, and functions in polar coordinates.	**MPAC 1:** Reasoning with definitions and theorems **MPAC 2:** Connecting concepts **MPAC 3:** Implementing algebraic/computational processes **MPAC 5:** Building notational fluency **MPAC 6:** Communicating
LO 2.2A: Use derivatives to analyze properties of a function.	**EK 2.2A1:** First and second derivatives of a function can provide information about the function and its graph including intervals of increase or decrease, local (relative) and global (absolute) extrema, intervals of upward or downward concavity, and points of inflection.	
LO 2.3B: Solve problems involving the slope of a tangent line.	**EK 2.3B1:** The derivative at a point is the slope of the line tangent to a graph at that point on the graph.	
LO 2.3C: Solve problems involving related rates, optimization, rectilinear motion, *(BC)* and planar motion.	**EK 2.3C4:** *(BC)* Derivatives can be used to determine velocity, speed, and acceleration for a particle moving along curves given by parametric or vector-valued functions.	
LO 3.3B(b): Evaluate definite integrals.	**EK 3.3B2:** If f is continuous on the interval $[a, b]$ and F is an antiderivative of f, then $\int_a^b f(x)\,dx = F(b) - F(a)$.	
LO 3.4C: Apply definite integrals to problems involving motion.	**EK 3.4C2:** *(BC)* The definite integral can be used to determine displacement, distance, and position of a particle moving along a curve given by parametric or vector-valued functions.	

To answer Question 1 successfully, students must apply the Mathematical Practices for AP Calculus as described below:

- Engage in **reasoning with definitions and theorems** (MPAC 1) when finding the total distance traveled.
- **Connect the concepts** (MPAC 2) of derivative and position of a particle as well as the concepts of vertical tangent lines and motion.
- Use **algebraic manipulation** (MPAC 3) to find $\frac{dy}{dx}$ and the equation of a tangent line.
- Build **notational fluency** (MPAC 5) by expressing $\frac{dy}{dx}$ in terms of $\frac{dy}{dt}$ and $\frac{dx}{dt}$ and in **communicating** (MPAC 6) the process that leads to finding the y-coordinate and the total distance.
- **Communicate** (MPAC 6) using accurate and precise language and **notation** (MPAC 5) in reporting information provided by technology and in explaining what the sign of $\left.\frac{dy}{dt}\right|_{t=3}$ implies about the vertical direction of motion of the particle.

Free Response: Section II, Part B

No calculator is allowed for problems on this part of the exam.

2. The function *f* has derivatives of all orders at $x = 0$, and the Maclaurin series for *f* is $\sum_{n=2}^{\infty} \frac{\ln n}{3^n n^3} x^n$.

   (a) Find $f'(0)$ and $f^{(4)}(0)$.

   (b) Does *f* have a relative minimum, a relative maximum, or neither at $x = 0$? Justify your answer.

   (c) Using the ratio test, determine the interval of convergence of the Maclaurin series for *f*. Justify your answer.

Learning Objectives	Essential Knowledge	Mathematical Practices for AP Calculus
LO 1.1A(b): Interpret limits expressed symbolically.	**EK 1.1A2:** The concept of a limit can be extended to include one-sided limits, limits at infinity, and infinite limits.	**MPAC 1:** Reasoning with definitions and theorems **MPAC 2:** Connecting concepts **MPAC 3:** Implementing algebraic/computational processes **MPAC 5:** Building notational fluency **MPAC 6:** Communicating
LO 2.2A: Use derivatives to analyze properties of a function.	**EK 2.2A1:** First and second derivatives of a function can provide information about the function and its graph including intervals of increase or decrease, local (relative) and global (absolute) extrema, intervals of upward or downward concavity, and points of inflection.	
LO 4.1A: Determine whether a series converges or diverges.	**EK 4.1A3:** Common series of numbers include geometric series, the harmonic series, and *p*-series.	
LO 4.1A: Determine whether a series converges or diverges.	**EK 4.1A5:** If a series converges absolutely, then it converges.	
LO 4.1A: Determine whether a series converges or diverges.	**EK 4.1A6:** In addition to examining the limit of the sequence of partial sums of the series, methods for determining whether a series of numbers converges or diverges are the *n*th term test, the comparison test, the limit comparison test, the integral test, the ratio test, and the alternating series test.	
LO 4.2A: Construct and use Taylor polynomials.	**EK 4.2A1:** The coefficient of the *n*th-degree term in a Taylor polynomial centered at $x = a$ for the function f is $\frac{f^{(n)}(a)}{n!}$.	
LO 4.2C: Determine the radius and interval of convergence of a power series.	**EK 4.2C2:** The ratio test can be used to determine the radius of convergence of a power series.	

To answer Question 2 successfully, students must apply the Mathematical Practices for AP Calculus as described below:

- Engage in **reasoning with the definition** (MPAC 1) of the coefficients of the Maclaurin series to find the coefficients for $f'(0)$ and $f^{(4)}(0)$ and in applying the ratio test and comparison test to determine convergence.
- **Connect the concepts** (MPAC 2) of the first and second derivative to find a relative minimum and the concepts of convergence and absolute convergence when finding the interval of convergence.
- Use **algebraic manipulation** (MPAC 3) including working with logarithms and functions to find specific coefficients in the Maclaurin series, the limit in the ratio test, and the interval of convergence.

- Display facility with **notation** (MPAC 5) in **communicating** (MPAC 6) the justification for why f has a relative minimum and what constitutes the interval of convergence.