Answers and Rubrics (BC)

Answers to Multiple-Choice Questions

1.	**C**
2.	**D**
3.	**C**
4.	**C**
5.	**B**
6.	**B**
7.	**A**
8.	**C**

Rubrics for Free-Response Questions

Question 1

Solutions	Point Allocation
(a) $\left.\frac{dy}{dx}\right\|_{t=3} = \left.\frac{dy/dt}{dx/dt}\right\|_{t=3} = \left.\frac{6\cos(1+\sin t)}{t-5\cos t}\right\|_{t=3} = 0.314$ An equation for the tangent line is $y = 2 + 0.314(x+1)$.	2 : { 1 : considers $\frac{dy}{dx}$ at $t = 3$ 1 : tangent line equation
(b) The tangent line is vertical when $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} \neq 0$. $\frac{dx}{dt} = 0$ when $t = 1.30644$. Because $y'(1.30644) = -2.305884 < 0$, the y-coordinate is decreasing and so the particle is moving down at that moment.	3 : { 1 : considers $\frac{dx}{dt} = 0$ 1 : $t = 1.30644$ 1 : conclusion with reason
(c) $y(3) = y(0) + \int_0^3 y'(t)\,dt$ $y(0) = y(3) - \int_0^3 y'(t)\,dt = y(3) + 1.63359 = 3.634$ (or 3.633)	2 : { 1 : Fundamental Theorem of Calculus 1 : answer
(d) Distance $= \int_0^3 \sqrt{(x'(t))^2 + (y'(t))^2}\,dt = 13.453$	2 : { 1 : integral 1 : answer

Question 2

Solutions	Point Allocation
(a) $\frac{f'(0)}{1!} = a_1 = 0 \Rightarrow f'(0) = 0$ $\frac{f^{(4)}(0)}{4!} = a_4 = \frac{\ln 4}{3^4 4^3} \Rightarrow f^{(4)}(0) = \frac{\ln 4}{3^4 4^3} \cdot 4! = \frac{\ln 4}{216}$	2 : { 1 : $f'(0)$ 1 : $f^{(4)}(0)$
(b) $f'(0) = 0$ $\frac{f''(0)}{2!} = a_2 = \frac{\ln 2}{3^2 2^3} \Rightarrow f''(0) = \frac{\ln 2}{3^2 2^3} \cdot 2! = \frac{\ln 2}{36} > 0$ By the Second Derivative Test, f has a relative minimum at $x = 0$.	2 : { 1 : considers $f''(0)$ 1 : answer with justification
(c) Using the ratio test, $\lim_{n\to\infty} \left\| \frac{\frac{\ln(n+1)}{3^{n+1}(n+1)^3} x^{n+1}}{\frac{\ln n}{3^n n^3} x^n} \right\| = \lim_{n\to\infty} \left\| \frac{\ln(n+1)}{\ln n} \cdot \left(\frac{n}{n+1}\right)^3 \cdot \frac{x}{3} \right\| = \left\| \frac{x}{3} \right\| < 1$ $\|x\| < 3$, therefore the radius of convergence is $R = 3$, and the series converges on the interval $-3 < x < 3$. When $x = 3$, the series is $\sum_{n=2}^{\infty} \frac{\ln n}{n^3}$. Because $0 < \frac{\ln n}{n^3} < \frac{n}{n^3} = \frac{1}{n^2}$ for all $n \geq 2$ and the p-series $\sum_{n=2}^{\infty} \frac{1}{n^2}$ converges, the series $\sum_{n=2}^{\infty} \frac{\ln n}{n^3}$ converges by the comparison test.	5 : { 1 : sets up ratio 1 : computes limit of ratio 1 : determines radius of convergence 1 : considers both endpoints 1 : analysis and interval of convergence

When $x = -3$, the series is $\sum_{n=2}^{\infty} (-1)^n \frac{\ln n}{n^3}$. This series is absolutely convergent because $\sum_{n=2}^{\infty} \frac{\ln n}{n^3}$ converges. The interval of convergence is $-3 \le x \le 3$.	